GEOPHYSICAL RESEARCH LETTERS, VOL. 19, NO. 16, PAGES 1691-1694, AUGUST 21, 1992

FAULT STABILITY UNDER CONDITIONS OF VARIABLE NORMAL STRESS

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Abstract. The stability of fault slip under conditions of varying normal stress is modeled as a spring and slider system with rate- and state-dependent friction. Coupling of normal stress to shear stress is achieved by inclining the spring at an angle, ϕ, to the sliding surface. Linear analysis yields two conditions for unstable slip. The first, of a type previously identified for constant normal stress systems, results in instability if stiffness is below a critical value. Critical stiffness depends on normal stress, constitutive parameters, characteristic sliding distance and the spring angle. Instability of the first type is possible only for velocity-weakening friction. The second condition yields instability if spring angle $\phi < -\cot^{-1}\mu_{ss}$, where μ_{ss} is steady-state sliding friction. The second condition can arise under conditions of velocity strengthening or weakening. Stability fields for finite perturbations are investigated by numerical simulation.

Introduction

The rate- and state-dependent representation of fault constitutive properties [Dieterich, 1979, 1981; Ruina, 1983] describes characteristic features of laboratory experiments including transient and steady-state velocity dependencies, time-dependent healing and displacement weakening at the onset of unstable slip. Applied to faults in nature, this formulation provides a framework for interpretation and modeling of various fault slip phenomena including stable creep, earthquake slip and afterslip [for example, Tse and Rice, 1986; Stuart, 1988; Marone *et al.*, 1991].

The spring-slider system adequately models the behavior of many laboratory experiments [Dieterich, 1981; Weeks and Tullis, 1985; Tullis and Weeks, 1986]. When rate- and state-dependence is incorporated, the onset of slip instability is associated with a critical stiffness:

$$K_c = \frac{\xi \sigma}{D_c} , \tag{1}$$

where σ is normal stress, D_c is a characteristic sliding distance appearing in rate- and state-dependent formulations and ξ is a parameter that depends on loading conditions and constitutive parameters [Dieterich, 1979, 1981; Ruina, 1983; Rice and Ruina, 1983, Rice and Gu, 1983; Gu et al., 1984; Weeks and Tullis, 1985; Blanpied and Tullis, 1986; Rice and Tse, 1986; Tullis and Weeks, 1986; Gu and Wong, 1991; Wong et al., 1992]. For stiffnesses $K < K_c$ unstable slip (stick-slip) will ensue, $K > K_c$ results in stable slip and at $K = K_c$ the system is neutrally stable.

A minimum patch size for unstable for unstable fault slip has been estimated by equating an elastic crack stiffness with equation (1):

$$l_c = \frac{G \eta}{K_c} = \frac{G \eta D_c}{\xi \sigma} , \tag{2}$$

where G is shear modulus (Poisson's ratio of 0.25), and η is a crack geometry parameter with values near 1 [Dieterich, 1986]. Hence, l_c may be interpreted to represent a minimum earthquake dimension. Numerical simulations of faults in two-dimensions further reveal that the zone of instability nucleation has the characteristic length l_c [Dieterich, 1992].

The previous stability studies assume constant normal stress. However, normal stress variations during slip are an integral component of many experiments and of faults in nature. For example, normal stress couples to shear stress in conventional triaxial sliding tests, though some servo-control schemes may minimize these effects at low sliding rates. In nature, normal stress will couple to slip whenever a fault is non-planar or the elastic medium is non-homogenous. Normal stress changes will also arise from fault interactions and where dip-slip faults interact with the free surface.

In this paper we examine conditions for unstable slip generalized to situations where normal stress is coupled to shear stress. We employ the rate- and state-dependent constitutive formulation for fault friction using the experimentally derived relation of Linker and Dieterich [1992] for dependence of state on normal stress history.

Linear Stability Analysis

Under conditions of fixed temperature the rate- and state-dependent constitutive framework may be expressed as

$$\tau = F\left(\dot{\delta}, \theta, \sigma\right) , \quad \dot{\theta} = G\left(\dot{\delta}, \theta, \sigma\right) , \tag{3}$$

where τ and σ are shear and normal stress respectively, θ is state and $\dot{\delta}$ is slip rate [Dieterich, 1979, 1981; Ruina, 1983; Chester and Higgs, 1992; Linker and Dieterich, 1992]. The following analysis employs the Ruina [1983] approximation of the Dieterich [1979] formulation for sliding resistance,

$$\tau = \sigma\left[\mu_o + A \ln(\dot{\delta} / \dot{\delta}^*) + B \ln(\theta / \theta^*) \right] , \tag{4}$$

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Paper number 92GL01821
0094-8534/92/92GL-01821$03.00

where μ_o, A and B are experimentally determined constants and $\dot{\delta}^*$ and θ^* are normalizing constants. Linker and Dieterich [1992] proposed the following equations to represent the evolution of state that results from variable slip rate and normal stress:

$$\frac{d\theta}{dt} = 1 - \left(\frac{\theta\dot{\delta}}{D_c}\right) - \left(\frac{\alpha\theta\dot{\sigma}}{B\sigma}\right) \tag{5}$$

or

$$\frac{d\theta}{dt} = -\left(\frac{\theta\dot{\delta}}{D_c}\right)\ln\left(\frac{\theta\dot{\delta}}{D_c}\right) - \left(\frac{\alpha\theta\dot{\sigma}}{B\sigma}\right) , \tag{6}$$

where α is a constant. Equation (6), absent the term for normal stress dependency and in different mathematical form, has been used in several studies (*e.g.*, Ruina [1983] and Tullis and Weeks [1986]). The two evolution laws have the property that θ evolves towards a steady-state value, $\theta_{ss} = D_c / \dot{\delta}$, over the characteristic slip, D_c. For silicates at room temperature, A and B generally have values of .005 to .01. If $B>A$, then steady state friction, μ_{ss}, decreases with increasing velocity. Linker and Dieterich [1992] established that α is positive and is therefore bounded above by μ_{ss}. They measured values in the range $0.25 \le \alpha \le 0.50$.

We consider the quasi-static behavior of the inclined-spring-block model, Figure 1, in which the shear traction supplied by the spring is balanced by frictional resistance

$$\tau = K\left(u - \delta\right) = F\left(\dot{\delta}, \theta, \sigma\right) . \tag{7}$$

Normal stress varies with displacement and shear stress, such that

$$\sigma = P - \tau \tan\phi , \tag{8}$$

where ϕ is spring angle and P is normal stress at $\tau = 0$. Figure 2 illustrates τ *vs.* σ loading paths from steady-state in relation to spring angle ϕ.

We examine the stability of steady-state sliding by performing a linearized analysis, following Ruina (1983). From (7) and (8) the steady-state sliding solutions for normal stress and slider displacement, δ, are

$$\sigma_{ss} = P - \tau_{ss}\tan\phi , \tag{9}$$

$$\delta_{ss} = V_0 t - \tau_{ss}/K , \tag{10}$$

where V_0 is the constant load point speed and τ_{ss} is evaluated using $\theta_{ss} = D_c / \dot{\delta}_{ss}$ and $\dot{\delta}_{ss} = V_0$.

For the linear analysis, new variables are introduced to represent variables relative to the steady-state sliding solution:

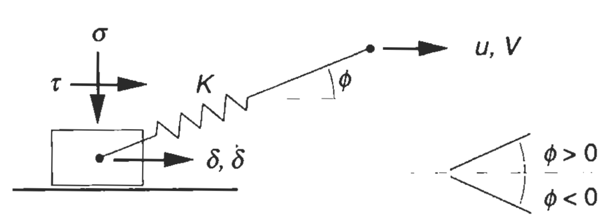

Fig 1. Inclined spring-slider model.

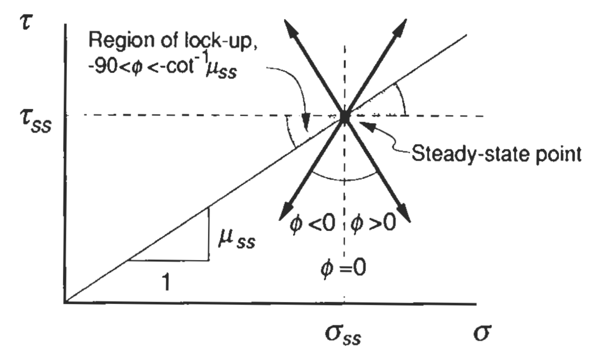

Fig 2. Loading paths (heavy lines) for perturbations from the steady-state friction point.

$$\hat{\tau} = \tau - \tau_{ss},\ \hat{\sigma} = \sigma - \sigma_{ss},\ \dot{\hat{\delta}} = \dot{\delta} - \dot{\delta}_{ss},\ \hat{\theta} = \theta - \theta_{ss}, \tag{11}$$

so that $\hat{\tau} = K\hat{\delta}$.

The constitutive relations (3) are then linearized taking

$$\hat{\tau} = \left(\frac{\partial F}{\partial\dot{\delta}}\right)\dot{\hat{\delta}} + \left(\frac{\partial F}{\partial\theta}\right)\hat{\theta} + \left(\frac{\partial F}{\partial\sigma}\right)\hat{\sigma} ,$$
$$\dot{\hat{\theta}} = \left(\frac{\partial G}{\partial\dot{\delta}}\right)\dot{\hat{\delta}} + \left(\frac{\partial G}{\partial\theta}\right)\hat{\theta} + \left(\frac{\partial G}{\partial\sigma}\right)\hat{\sigma} . \tag{12}$$

The partial derivatives in (12) are evaluated at steady-state, using the fact that $(\dot{\delta} - V_0) = 0$. Thereafter, the partial derivatives are considered as constants. The variable $\hat{\sigma}$ is eliminated via the inclined-spring-slider geometry:

$$\hat{\sigma} = K\hat{\delta}\tan\phi . \tag{13}$$

This yields a coupled pair of linear equations for $d\hat{\delta}/dt$ and $d\hat{\theta}/dt$ in terms of $\hat{\delta}$ and $\hat{\theta}$ with constant coefficients:

$$\frac{d\hat{\delta}}{dt} = a_{11}\hat{\delta} + a_{12}\hat{\theta} ,$$
$$\frac{d\hat{\theta}}{dt} = a_{21}\hat{\delta} + a_{22}\hat{\theta} , \tag{14}$$

where

$$a_{11} = -\frac{K\dot{\delta}_{ss}(1 + \mu_{ss}\tan\phi)}{A\sigma_{ss}} ,$$
$$a_{12} = -\frac{B\dot{\delta}_{ss}}{A\theta_{ss}} ,$$
$$a_{21} = \left[\frac{\theta_{ss}\dot{\delta}_{ss}K}{AD_c\sigma_{ss}}\right]\left[1 + \left(\frac{\alpha D_c}{B\sigma_{ss}}\right)K\tan\phi\right]\left[1 + \mu_{ss}\tan\phi\right] ,$$
$$a_{22} = -\left[\frac{\dot{\delta}_{ss}}{D_c}\right]\left[1 - \left(\frac{B}{A}\right)\left(1 + \left(\frac{\alpha D_c}{B\sigma_{ss}}\right)K\tan\phi\right)\right] . \tag{15}$$

At steady state the two proposed evolution laws, (5) and (6), yield identical expressions for $d\hat{\theta}/dt$. Hence, coefficients of the linear equations (15) are the same whether one adopts (5) or (6). This system has solutions of the form

$$\hat{\delta} = Re\left(A_1 e^{st}\right),$$
$$\hat{\theta} = Re\left(A_2 e^{st}\right), \qquad (16)$$

where $Re(\ )$ indicates the real part of the expression. To evaluate the solution, (16) is substituted into (14) and use is made of the derivatives from (16), $d\hat{\delta}/dt = A_1 s e^{st}$ and $d\hat{\theta}/dt = A_2 s e^{st}$. These substitutions result in a quadratic equation in s:

$$s^2 + bs + c = 0, \qquad (17)$$

where $b = -(a_{11}+a_{22})$ and $c = a_{11}a_{22}+a_{12}a_{21}$.

From (16), if $Re(s) > 0$, then perturbations of slip and slip rate grow exponentially, *i.e.* instability results. This condition gives two stability boundaries corresponding to a critical stiffness, K_c, and a geometrical instability.

The stability boundary at $K = K_c$ corresponds to the condition $b = -(a_{11}+a_{22}) = 0$ when $c \geq 0$. Evaluating $a_{11}+a_{22} = 0$, gives the critical stiffness

$$K_c = \frac{\sigma_{ss}(B - A)}{D_c\left[1 + (\mu_{ss} - \alpha)\tan\phi\right]}. \qquad (18)$$

At $K = K_c$ a perturbation induces stable oscillations. If $b^2 < 4c$, $c > 0$ and $b > 0$, then oscillations decay. If $b^2 < 4c$, $c > 0$ and $b < 0$, then oscillations grow exponentially. If, $b^2 > 4c$, $c > 0$, and $b < 0$, then perturbations grow monotonically. See Figure 3, points a, b and c.

The second stability boundary exists along $c = (a_{11}a_{22}+a_{12}a_{21}) = 0$ when $b \leq 0$. This second boundary does not exist under conditions of constant normal stress, since $(a_{11}a_{22}+a_{12}a_{21})$ is then always positive (Ruina, 1983). Evaluation of $(a_{11}a_{22}+a_{12}a_{21}) = 0$, reduces to

$$\phi = -\cot^{-1}\mu_{ss}. \qquad (19)$$

This boundary is purely a geometric effect and corresponds to loading paths along the steady-state friction curve. If $\phi < -\cot^{-1}\mu_{ss}$, then loading paths have a flatter slope than the friction curve. Under that condition, perturbations from steady-state that decrease τ result in runaway slip, while those that increase τ cause the slider to lock (Figure 2). A loading path from zero initial stress also causes the slider to lock and so the system cannot reach steady state.

Numerical Simulation

Numerical simulations have been performed to examine some characteristics of finite amplitude deviations from steady-state. The model consists of a spring and slider as illustrated by Figure 1. For these simulations, the evolution relation of (6) was used. The spring angle is inclined at a fixed angle ϕ. Linker and Dieterich [1992] describe the time-marching numerical procedure used for these calculations.

Finite amplitude perturbations from steady state can result in unstable slip even when K is somewhat greater than K_c as defined by equation (18) (Figure 3, point d). The mapping of stability fields by stiffness and amplitude is qualitatively similar to constant normal stress results [Gu et al., 1984].

Discussion

Our critical stiffness condition for instability generalizes the previous result of equation (1). Under constant normal stress $\xi = B - A$ for perturbations from steady state [Ruina, 1983; Rice and Ruina, 1983]. When shear stress and normal stress are coupled through spring angle ϕ then $\xi = (B - A)/[1 + (\mu_{ss} - \alpha)\tan\phi]$. Because $0 \leq \alpha \leq \mu_{ss}$ [Linker and Dieterich, 1992] an increase in negative spring angle results in an increase in critical stiffness and so tends to destabilize slip.

Linker and Dieterich [1992] argued that in the inherently unstable situation of decreasing normal stress, the state evolution parameter α should have a stabilizing effect relative to the case $\alpha = 0$. Equation (18) confirms their conclusion. When spring angle is negative, values of $\alpha > 0$ yield smaller

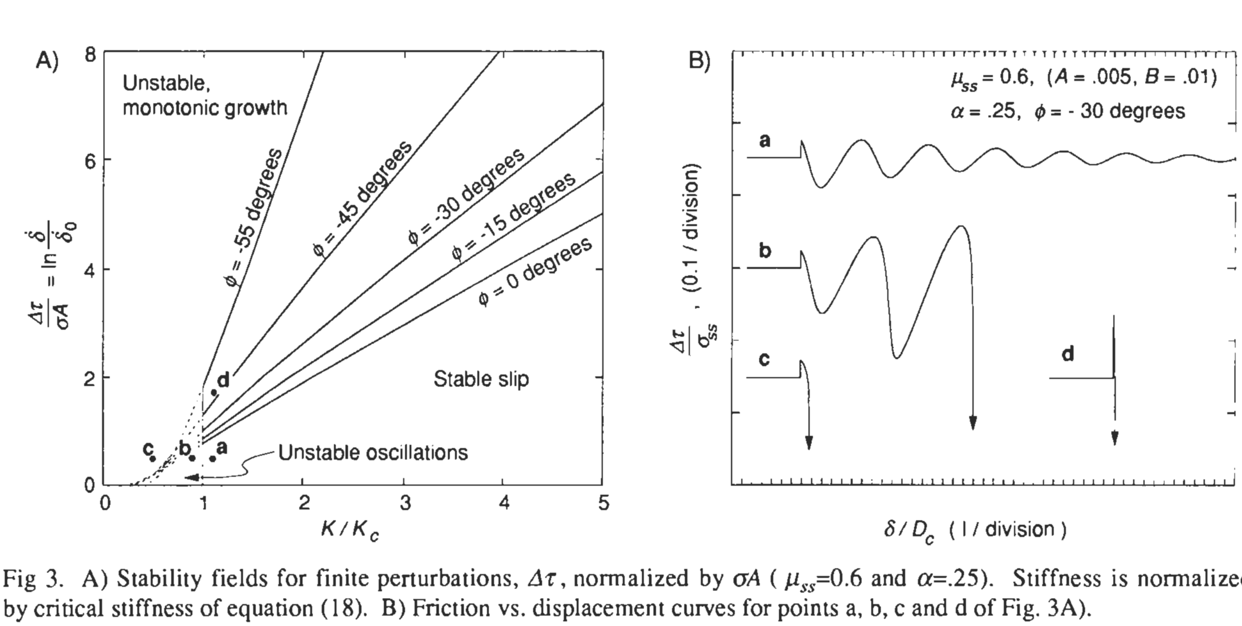

Fig 3. A) Stability fields for finite perturbations, $\Delta\tau$, normalized by σA (μ_{ss}=0.6 and α=.25). Stiffness is normalized by critical stiffness of equation (18). B) Friction vs. displacement curves for points a, b, c and d of Fig. 3A).

K_c than for $\alpha = 0$. The reverse holds for positive spring angles.

The second condition for instability (19) does not arise in constant normal stress systems. This condition is independent of stiffness, normal stress and constitutive parameters except as they determine μ_{ss}. For usual friction values of μ_{ss}~0.6, the critical angle is approximately -60 degrees.

Laboratory experiments that employ an inclined fault and fixed minimum principal stress, *e.g.* the triaxial test, are represented by taking ϕ as the negative of the angle between the principal loading axis and the fault. The negative spring angle enhances the tendency for unstable slip since it increases K_c. However, instability as a perturbation from steady-state is possible only under velocity-weakening conditions, since (18) yields positive critical stiffnesses only if $(B\text{-}A)>0$, as found previously for constant normal stress. The second instability condition is independent of $(B\text{-}A)$ and may occur with velocity strengthening or velocity weakening. Because the critical spring angle of the second instability condition is generally about -60 degrees, instability of this type may not be important in laboratory experiments where sliding surfaces are generally inclined at angles of 45 degrees or less. However, this condition might arise on faults that develop high angle bends or kinks.

In nature, normal stress will couple to slip in a variety of circumstances outlined previously. Both instability conditions have the property that negative spring angles are less stable than positive angles. Negative spring angle means that fault slip and load point displacement lead to changes of both shear stress and normal stress of the same sign.

Acknowledgements. We thank M. Blanpied, P. Segall and J. Weeks for reviews and J. R. Rice for discussion and comment. Linker received support from NEHRP grant number 1788 to Harvard University.

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(Received: June 9, 1992;
accepted: June 25 1992).